Spectral theory of a class of canonical systems with two singular endpoints

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We consider 2×2-Hamiltonian systems without potential:

$$y'(t) = zJH(t)y(t), \quad t \in (a, b). \tag{1}$$

Here the *Hamiltonian* H shall be a locally integrable function which takes real and nonnegative 2×2-matrices as values and does not vanish on a set of positive measure. Moreover, $z \in \mathbb{C}$ is the spectral parameter, and J denotes the signature matrix $J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. The equation (1) is the eigenvalue equation of a differential operator; our aim is to investigate the spectral theory of its selfadjoint realisations.

Equations of the form (1) occur in many situations. For example, the theory of canonical systems naturally includes Sturm-Liouville- (in particular Schrödinger-) equations, Jacobi matrices, Dirac systems, etc.

We consider a certain class of Hamiltonians with two singular endpoints, meaning that at both endpoints Weyl's limit point case prevails. Our aim is to develop a spectral theory along the lines of Weyl's theory for the regular (limit circle) case. This includes:

- Definition of a Titchmarsh-Weyl coefficient (also called M-function) and a spectral measure.
- Establishing direct spectral theorems in the form of showing properties of the Titchmarsh-Weyl coefficient and the spectral measure.
- Stating and establishing inverse spectral theorems in the form of showing existence and uniqueness theorems.

The class of Hamiltonians under consideration is sufficiently narrow to allow for complete and satisfactory answers. In the same time it is sufficiently wide to include several interesting examples, for instance perturbed Bessel-operators.

Our methods rely on Pontryagin space theory; we take a detour through the indefinite world to arrive at results for the described classical problem.

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