Dual addition formula for Gegenbauer polynomials

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Abstract.

The addition formula for Legendre polynomials $P_n(x)$ gives the expansion of $P_n(\cos x \cos y + \sin x \sin y \cos t)$ in terms of $\cos(kt)$ $(k = 0, 1, \ldots, n)$. Its constant term implies the product formula

$$P_n(\cos x)P_n(\cos y) = \pi^{-1} \int_0^\pi P_n(\cos x \cos y + \sin x \sin y \cos t)dt.$$

A dual version of this product formula is the linearization formula

$$P_m(x)P_n(x) = \sum_{k=0}^{\min(m,n)} c_{m,n,k} P_{m+n-2k}(x).$$

Askey conjectured that there is a related dual addition formula which expands $P_{m+n-2k}(x)$ in terms of certain functions of k. The lecture will answer this in the positive sense, also more generally for Gegenbauer polynomials. The needed functions of k are special Racah polynomials.
Reference: arXiv:1607.06053