Global well-posedness of the Cauchy problem for the mass-subcritical NLS with initial data in a modulation space

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Abstract. In this talk the global well-posedness of the mass-subcritical nonlinear Schrödinger (NLS) equation in \mathbb{R}^d for initial data in the modulation space $M_{p,p'}(\mathbb{R}^d)$ with p > 2 sufficiently close to 2 is presented. The proof is based on Bourgain's high-low frequency decomposition method. Additionally, some related results will be mentioned.

The NLS is commonly used to model the signal propagation in nonlinear optical fibers. The mathematical theory of the NLS is well-developed in $L^2(\mathbb{T})$ (periodic signals) and in $L^2(\mathbb{R})$ (decaying signals). However, signals transporting information are neither decaying nor are they periodic.

The study of such signals can be performed in the modulation space $M_{\infty,q}(\mathbb{R})$. Modulation spaces $M_{p,q}(\mathbb{R}^d)$ are defined in terms of the short-time Fourier transform and, roughly speaking, the parameters p and q correspond to the decay in the space and frequency domains respectively. As $L^2(\mathbb{R}) = M_{2,2}(\mathbb{R})$, the aforementioned result can be seen as a step towards the non-decaying signals.

References

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